Rockwell SOC team

Sequential Simplex Optimization

Engineering Review Board

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Purpose

- Contrast single-factor-at-a-time and multi-factor optimization.
- Contrast random number dependent (Monte Carlo) and multi-factor optimization.
- Introduce the Sequential Simplex Optimization scheme.
  - Fixed-step Simplex.
  - Variable-step Simplex.

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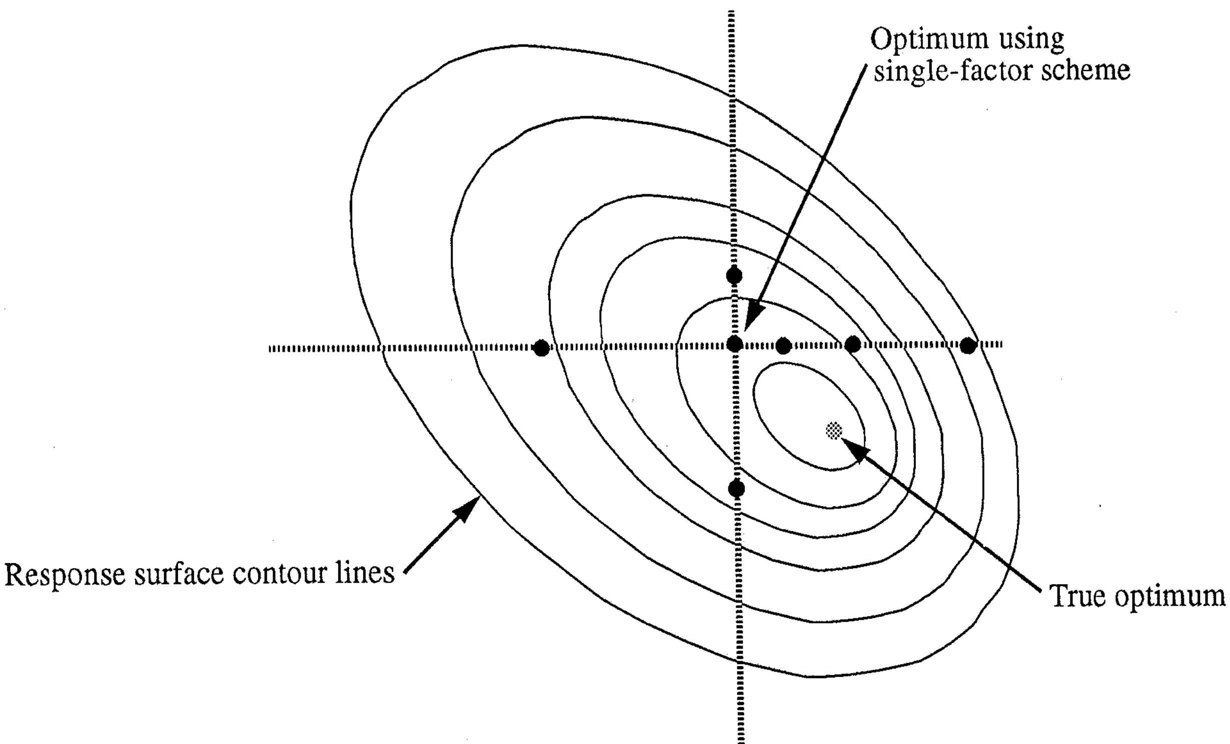
Background

- Single-factor-at-a-time optimization.
  - •• Currently used by FDD trajectory simulations.
    - ••• STAMPS
    - ••• DIBS
  - •• This scheme optimizes one factor while holding all others fixed.
- Optimizing one factor at a time requires the optimization loops to be nested.
  - •• This nesting causes a geometrical growth in the number of experiments.
- Monte Carlo optimization requires a sufficient number of experiments to adequately cover the factor space.
  - •• Limited control over sampling domain.
  - •• A "sufficient" number of experiments is somewhat dependent on the system being studied.

Background (Con't.)

- Single-factor-at-a-time optimization can miss finding the optimum.
  - Cannot take into account *factor interaction.*
  - This example shows a solution stranded on an oblique ridge of the response surface.



Simplex Algorithm

- The Sequential Simplex algorithm uses what is referred to as *evolutionary operation* (EVOP).
- Simplex adjusts **all** factors per iteration.
  - No nesting of iteration loops.
  - This results in a drastically smaller case growth as a function of the number of factors.
  - The number of experiments typically is 5 to 10 times the number of factors
  - Example: a four factor optimization problem will require from 20 to 40 experiments.
- Comparing the number of experiments needed for sequential simplex and single-factor-at-a-time:

Number of Factors	Sequential Simplex	Single Factor
2	6 - 12	4 - 9
4	20 - 30	16 - 81
8	70 - 100	256 - 6561
16	120 - 160	60000+

How Simplex Optimization Works

- All factors are adjusted per iteration.
- The worst experiment is rejected and replaced by a projection directed along an approximated response surface gradient.
- There are two algorithms: fixed-step and variable-step.
  - Fixed-Step:
    - The progression along the response surface is in fixed-distance units.
    - Best used when maximum control is needed over the experiments.
  - Variable-step:
    - Allows for faster traversal of steep areas of the response surface.
    - Also will contract as the experiment points near the optimum.

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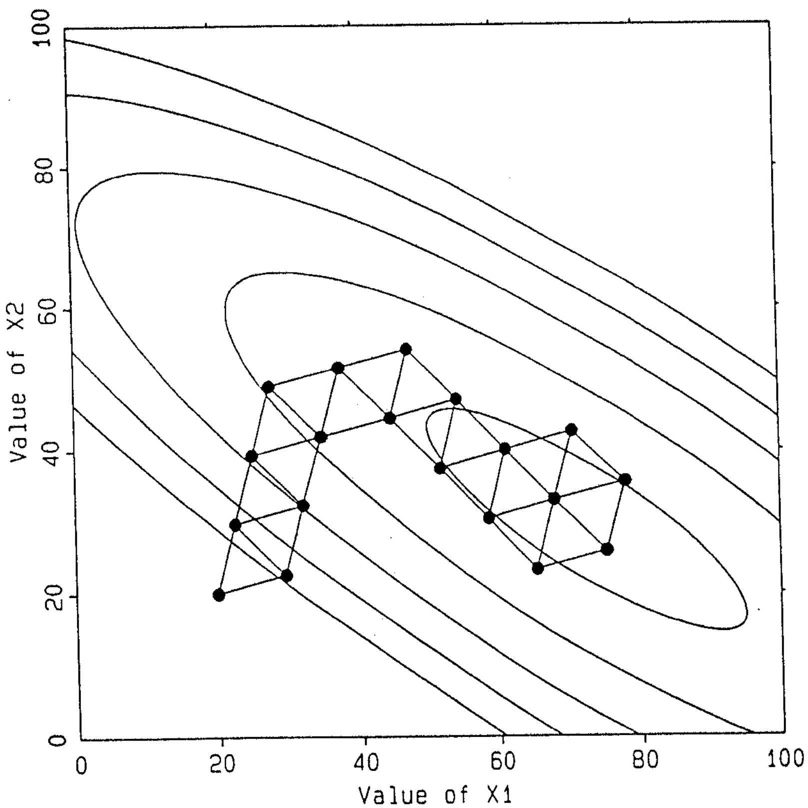
How Simplex Optimization Works (Con't)

- Fixed-step Simplex example:



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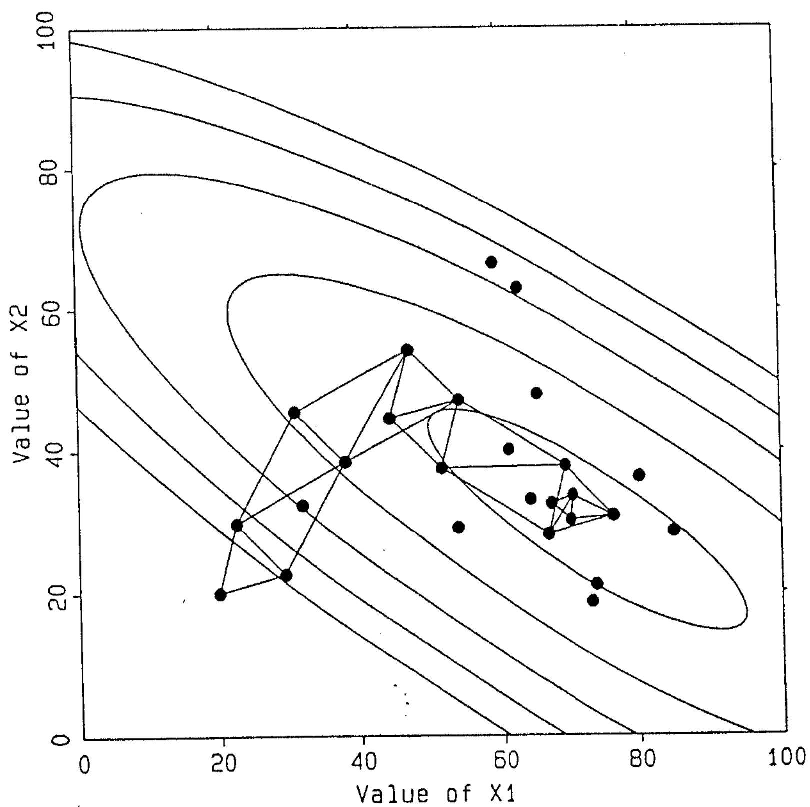
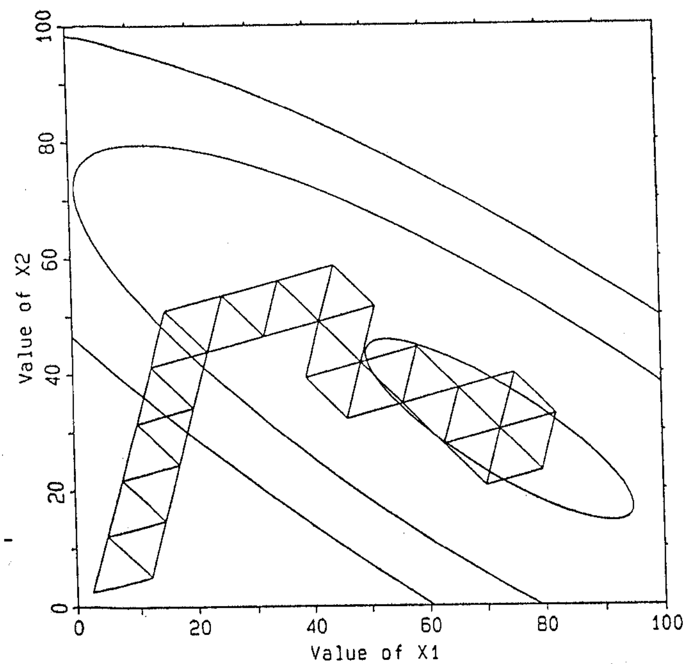
How Simplex Optimization Works (Con't)

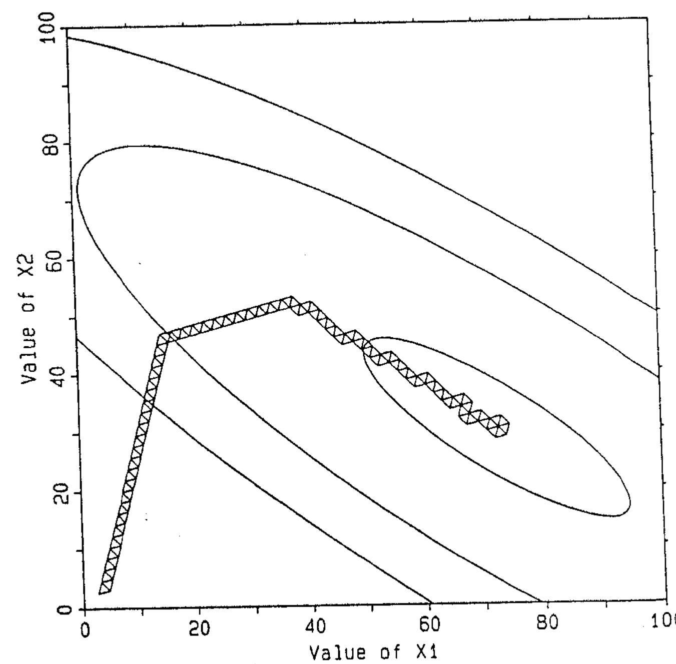
- Variable-step Simplex example:



How Simplex Optimization Works (Con't)

- Fixed-step Simplex step size behavior:





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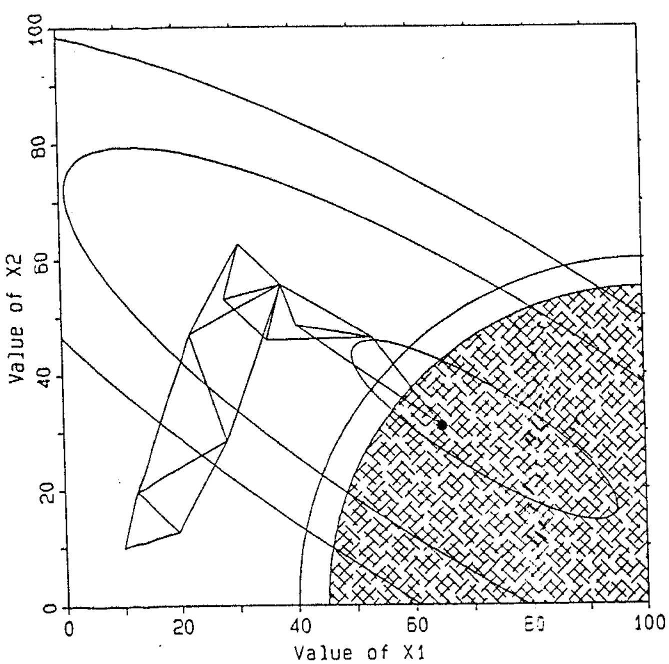
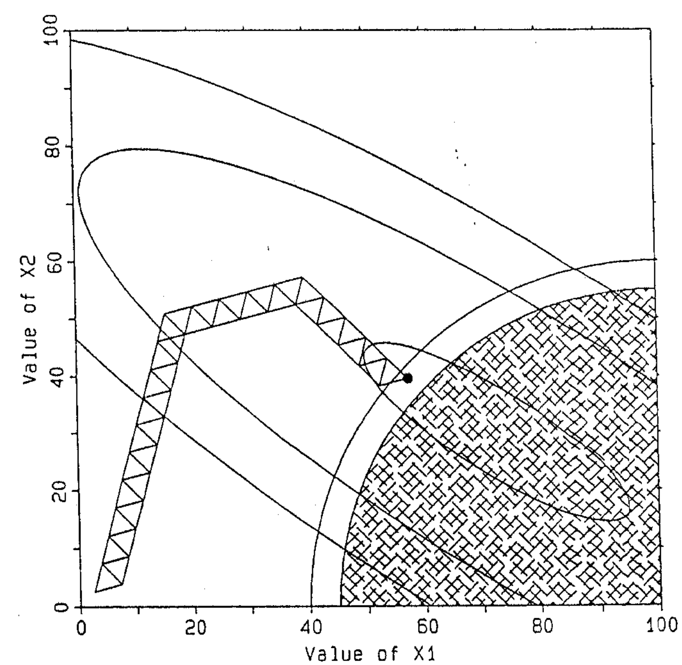
How Simplex Optimization Works (Con't)

- Variable-step Simplex danger: expanding into an explosive region.





How Simplex Optimization Works (Con't)

- Worksheets have been designed to simplify the Simplex process.
- The algorithm lends itself easily to coding in a computer.
- Fixed-step algorithm:

1. Rank the experiments of the initial simplex on a worksheet in decreasing order from best to worst. Put the worst experiment into the row labeled **W**.

2. Calculate and evaluate **R**.

3. *Never* transfer the current row labeled **W** to the next worksheet. *Always* transfer the current row labeled **N** to the row labeled **W** on the next worksheet. Rank the remaining retained experiments in order of decreasing response on the new worksheet, and go to step 2.

Variable-step algorithm

1. Rank the experiments of the initial simplex on a worksheet in decreasing order from best to worst. Put the worst experiment into the row labeled **W**.

2. Calculate and evaluate **R**:
   A. If **N** <= **R** <= **B**, use simplex **B..NR**, and go to 3.
   B. If **R** > **B**, calculate and evaluate **E**:
      i. If **E** >= **B**, use simplex **B..NE**, and go to 3.
      ii. If **E** < **B**, use simplex **B..NR**, and go to 3.
   C. If **R** < **N**:
      i. If **R** >= **W**, calculate and evaluate $\mathbf{C}_R$, use simplex $\mathbf{B..NC}_R$, and go to 3.
      ii. If **R** < **W**, calculate and evaluate $\mathbf{C}_W$, use simplex $\mathbf{B..NC}_W$, and go to 3.

3. *Never* transfer the current row labeled **W** to the next worksheet. *Always* transfer the current row labeled **N** to the row labeled **W** on the next worksheet. Rank the remaining retained experiments in order of decreasing response on the new worksheet, and go to step 2.

How Simplex Optimization Works (Con't)

- Fixed-step worksheet:

Simplex No. ___ → ___	Factor					
	x_1	x_2	Response	Rank	Vertex Number	Times Retained
Coordinates of				B		
retained vertexes				N		
Σ						
$\bar{P} = \Sigma/k$						
W				W		
$(\bar{P} - W)$						
$R = \bar{P} + (\bar{P} - W)$				R		0

Variable-step worksheet:

Simplex No. ___ → ___	Factor					
	x_1	x_2	Response	Rank	Vertex Number	Times Retained
Coordinates of				B		
retained vertexes				N		
Σ						
$\bar{P} = \Sigma/k$						
W				W		
$(\bar{P} - W)$						
$R = \bar{P} + (\bar{P} - W)$				R		0
$(\bar{P} - W)/2$						
$C_w = \bar{P} - (\bar{P} - W)/2$				C_w		0
$C_r = \bar{P} + (\bar{P} - W)/2$				C_r		0
$E = R + (\bar{P} - W)$				E		0

Using the Simplex Algorithm to Solve Problems

- The Simplex method is not limited to optimization problems.
- Simplex can be used for maxima, minima, or boundary problems.
  - Example: Determine what sigma level corresponds to a set of conditions (ET disposal footprint, entry cross-range, etc.).
- The "response" that is used in the algorithm can be a combination of a series of individual responses, factored together using desirability functions.
  - Example: ascent first stage optimization.
    - Factor examples are SAR pitch and yaw, and throttle down and throttle up times.
    - Constraint examples are Q-overshoot, staging hdot, SRB apogee, and throttle-altitude.
  - The vehicle performance (gross response) can be weighted by how the trajectory conforms to the various constraints.
  - This example also illustrates a problem with a prohibitively large dimensionality.
- Desirability functions are very similar to member functions in fuzzy logic.

Conclusions and Recommendations

- Sequential simplex optimization can be used for multi-factor optimization problems to reduce the required number of experiments.
  - The reduction is not evident for two- or three-factor systems.
- The simplex algorithm is an evolutionary operation, learning from earlier results to improve subsequent experiments.
  - This differs from Monte Carlo optimization.
- Sequential Simplex is widely used in industry.
  - Applications range from chemical manufacturing to power generation to spectral analysis.
  - Monte Carlo optimization is rarely used.
- Next Step: evaluate Sequential Simplex as a possible Monte Carlo replacement for certain tasks performed in Flight Design.