Rockwell SOC team

Multi-Stage Monte Carlo Optimization

Engineering Review Board

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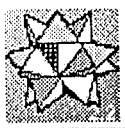
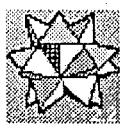


Purpose & Scope

- This presentation will introduce Multi-Stage Monte Carlo optimization and an Enhanced Multi-Stage Monte Carlo algorithm.

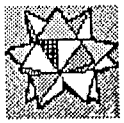
- The Enhanced Multi-Stage Monte Carlo and regular Multi-Stage Monte Carlo approaches will be contrasted with the traditional Monte Carlo approach using an example optimization problem.

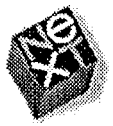
- Several applications use Monte Carlo algorithms.
  - DADS
  - Descent SVDS/STAMPS
  - Rendezvous/Prox Ops.



What is Monte Carlo Optimization?

- Monte Carlo optimization evaluates a "function" (e.g., a process or an equation) over a factor space selecting evaluation points at random. There are (at least) two methods of determining when the optimum has been found:
  - Several sets of runs are made to insure a consistent answer. If the variance in the answers exceeds a certain tolerance, then the sampling density is increased.
  - Optimization runs are made at higher and higher sampling densities until the answers agree to within a certain tolerance.
- Sampling of the factor space is employed rather than stepping through the factor space systematically since for many-variable problems the number of points becomes prohibitively large.





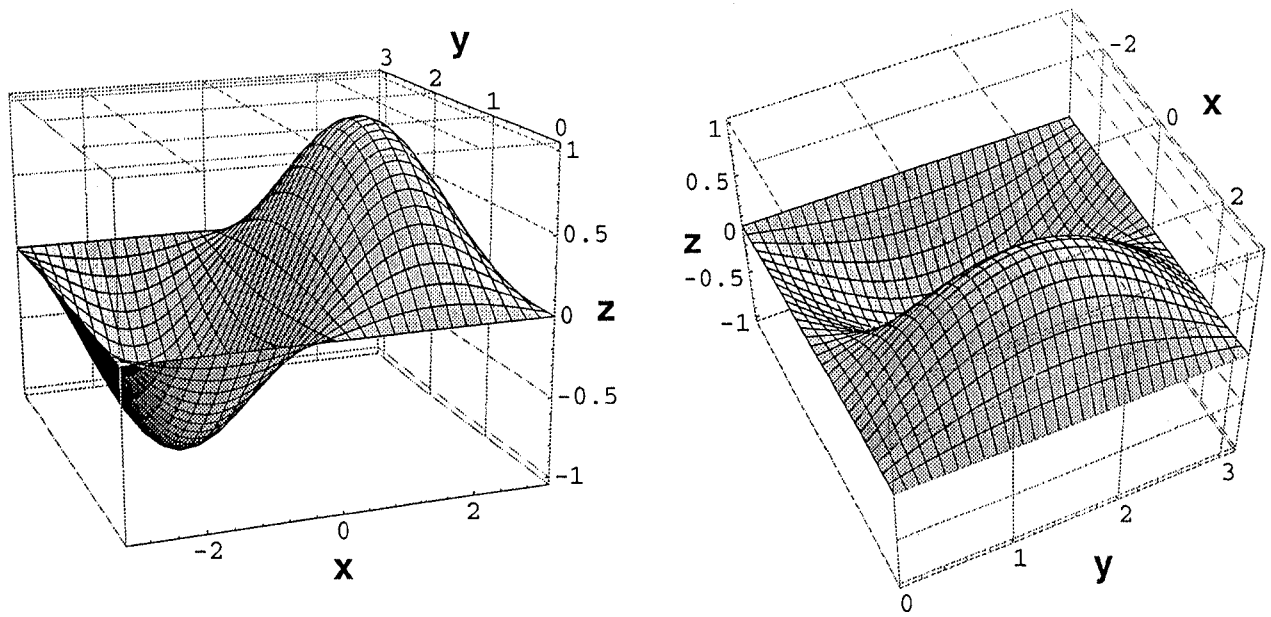
What is *Multi-Stage* Monte Carlo?

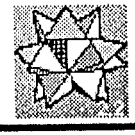
- Multi-Stage Monte Carlo (Conley, 1981) adds a level of intelligence to the search by periodically re-focusing the factor space around the "current optimum."
- After a selected number of points are evaluated, the size of the factor space is reduced and centered around the current "best response" point.
- The advantage to this is that the algorithm can spend more time searching the area near the true optimum, while spending less time in areas far from the optimum.
- The disadvantage is that the search can become distracted by a local optimum and miss the global optimum. Allman (1995) has suggested an improvement to minimize this disadvantage.

Example Problem

- We illustrate using the Monte Carlo, Multi-Stage Monte Carlo and Enhanced Multi-Stage algorithms by optimizing the function

$$z = \text{Sin}[x]\ \text{Sin}[y], \quad \text{where} \quad -\pi <= x <= \pi$$
$$0 <= y <= \pi$$





Example Problem (Con't)

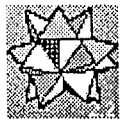
- For each method we will calculate a total of 1000 points, searching the factor space for a maximum response.

- The final optimum and a scatter plot of points will be presented to show how the Multi-Stage and Enhanced Multi-Stage algorithms concentrate the search.

- We calculate the optimum factor point analytically (partial derivatives set to 0) to be

$$x = y = {}^{\pi}/_{2} = 1.57079632679$$

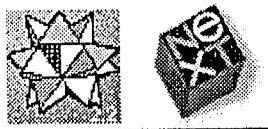
with a response (Sin[π/2] Sin[π/2]) of 1.0.



Example Problem -- Monte Carlo Solution

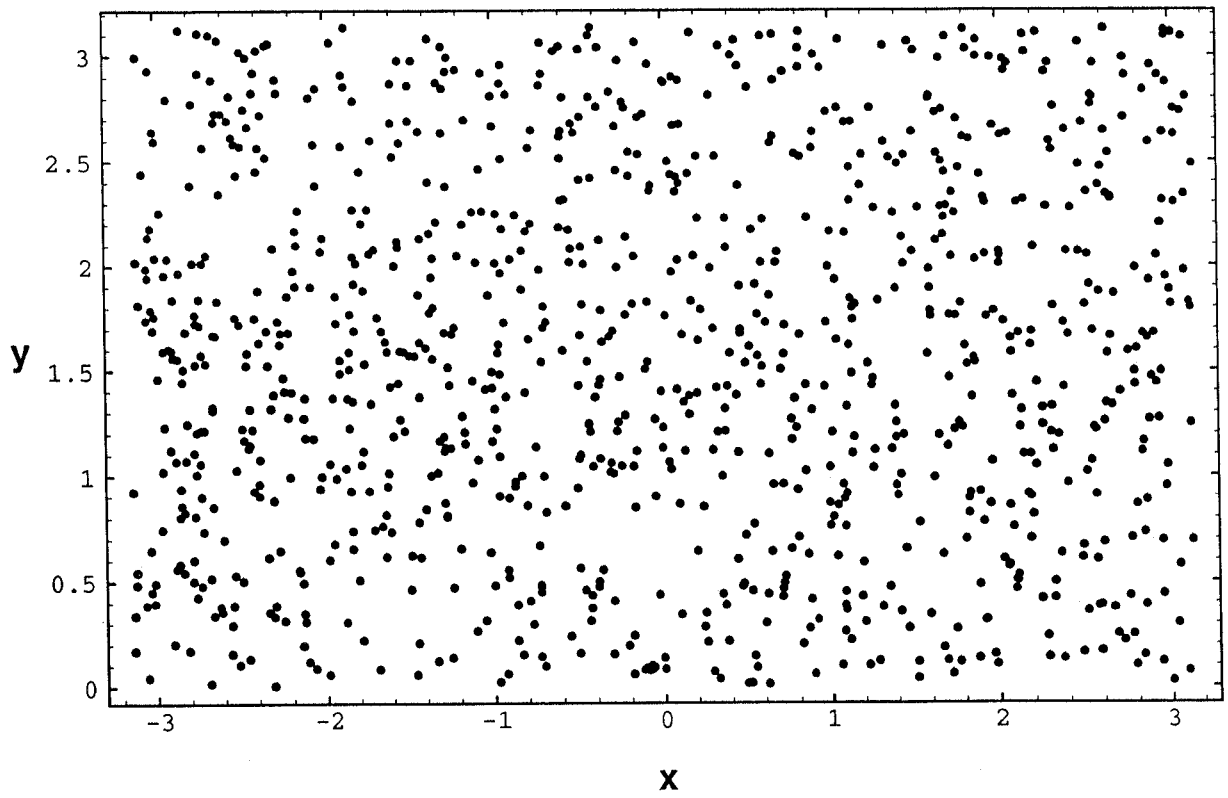
- We list four results of the algorithm.

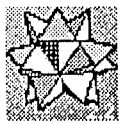
```
Max factor point (1.50201992,1.56866337), response 0.997633566
Max factor point (1.66907722,1.56304846), response 0.995144449
Max factor point (1.60483374,1.48394327), response 0.99565361
Max factor point (1.58229817,1.57738585), response 0.999912145

Mean:       (1.58955726,1.54826024),          Mean: 0.99708157

Std. Dev.: (0.06897409,0.04328184),      Std. Dev.: 0.00217248
```



Example Problem -- Monte Carlo Solution (Con't)

- The following is a scatter plot showing the 1000 sample points.

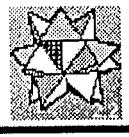




Example Problem -- Multi-Stage Monte Carlo Solution

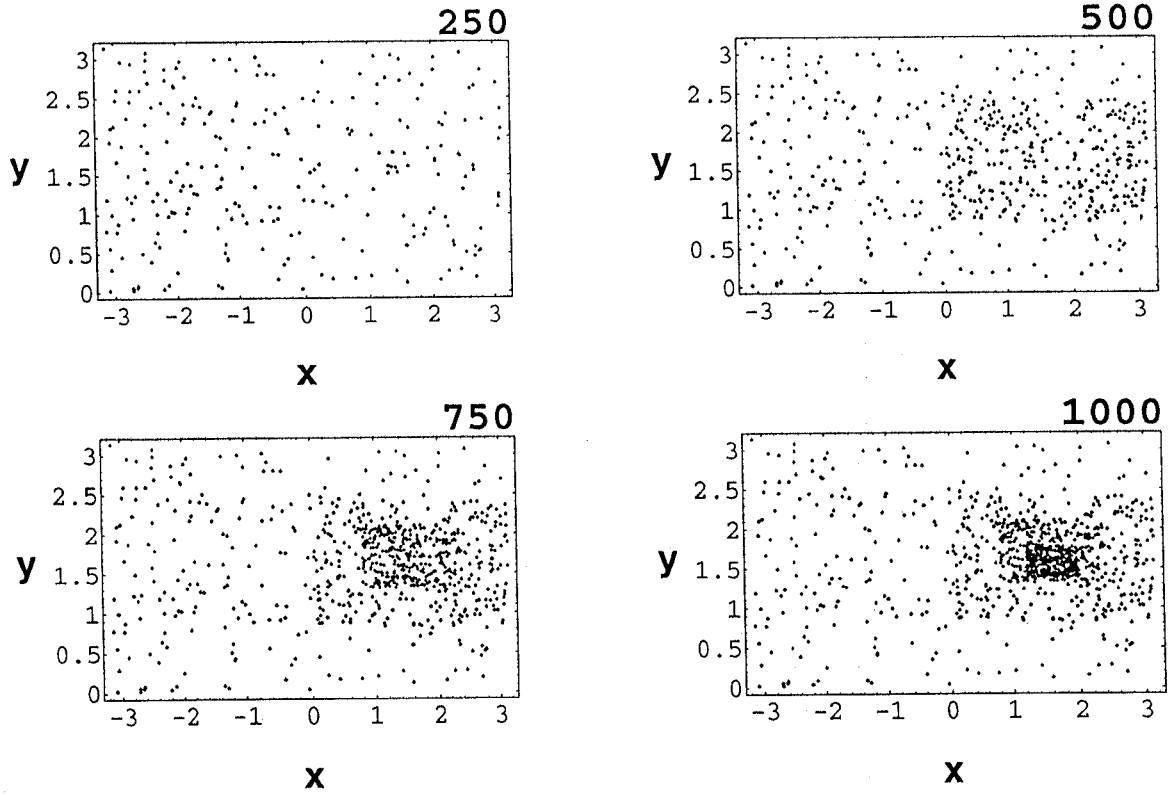
- We break up the 1000 points into four sets of 250.
- Each set of points is evaluated across a contracted factor space.
- The size of the factor space is reduced by 1/2 per set, so the last set spans 1/64 of the original space.
- We again list four results of the algorithm.

```
Max factor point (1.56320706,1.56257135), response 0.999937378
Max factor point (1.57176664,1.60184902), response 0.999517433
Max factor point (1.55846449,1.54762896), response 0.999655633
Max factor point (1.57730479,1.54662022), response 0.999686598

Mean:       (1.56768575,1.56425388),          Mean: 0.99969926

Std. Dev.: (0.00845125,0.02623374),     Std. Dev.: 0.00017533
```



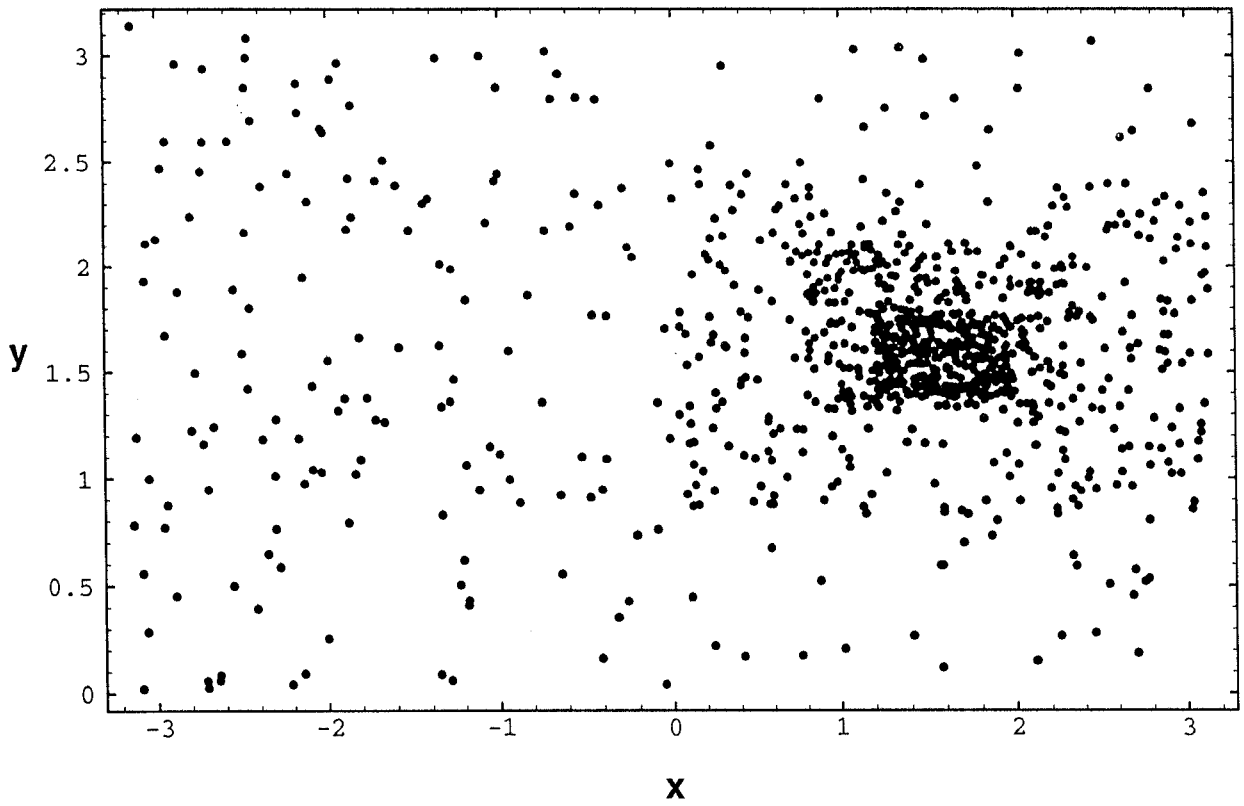
Example Problem -- Multi-Stage Monte Carlo Solution (Con't)

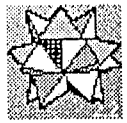
- Here is the progression of sampling through the four steps, showing the contraction of the factor space.



Example Problem -- Multi-Stage Monte Carlo Solution (Con't)

- Here is the final plot in full size.

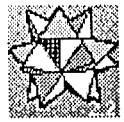




Example Problem -- Enhanced Multi-Stage Monte Carlo Solution

- A modification to the Multi-Stage Monte Carlo algorithm is currently being evaluated.
- The change increases the sample size in the early iterations and decreases the sample size in the later iterations.
- This change provides a greater sampling over the larger factor space where the function being optimized has (potentially) more variability.
- For this example problem, the sampling density is:

First iteration:	533 points
Second iteration:	266 points
Third iteration:	133 points
Fourth iteration:	66 points



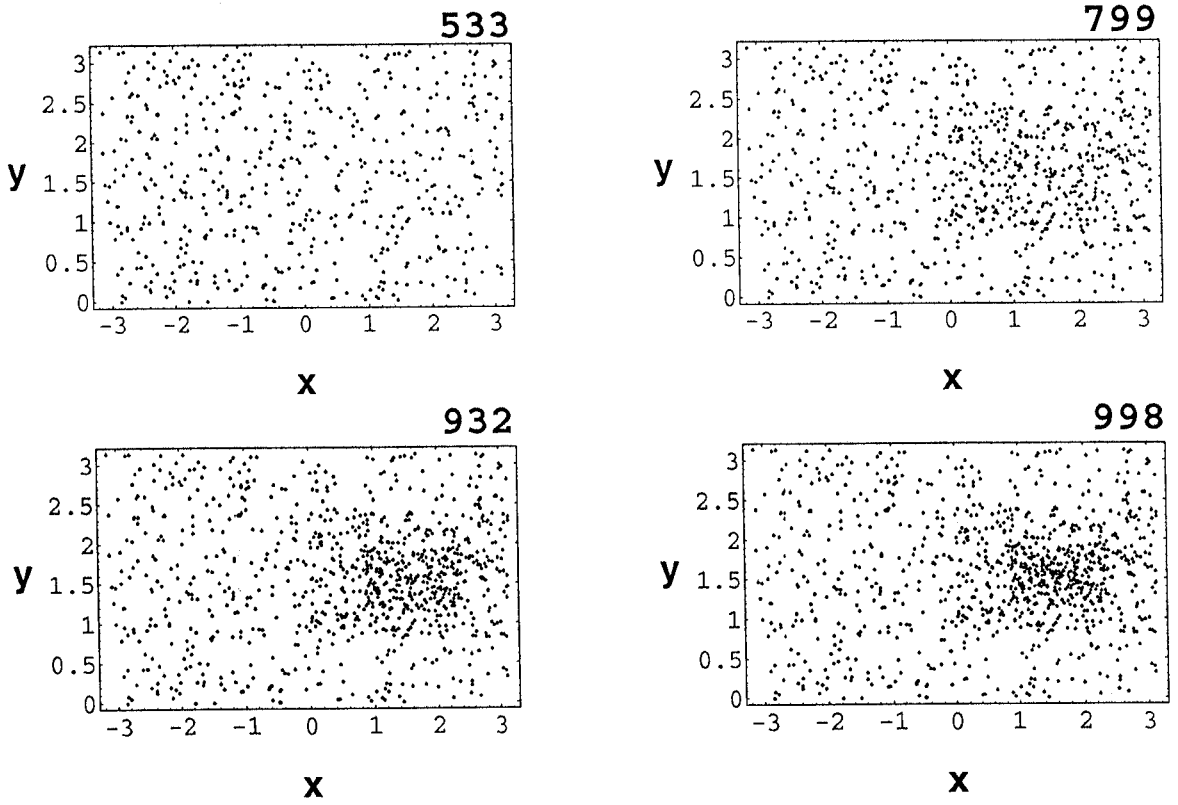
Example Problem -- Enhanced Multi-Stage Monte Carlo Solution (Con't)

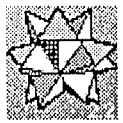
- We again list four results of the algorithm

```
Max factor point (1.55295453,1.55622140), response 0.999734644
Max factor point (1.56848883,1.57747045), response 0.999975066
Max factor point (1.58611469,1.58329718), response 0.999804551
Max factor point (1.61023794,1.56991021), response 0.999221888

Mean:       (1.57944900,1.57172481),          Mean: 0.99968404
Std. Dev.: (0.02459317,0.01169873),     Std. Dev.: 0.00032387
```

Example Problem -- Enhanced Multi-Stage Monte Carlo Solution (Con't)

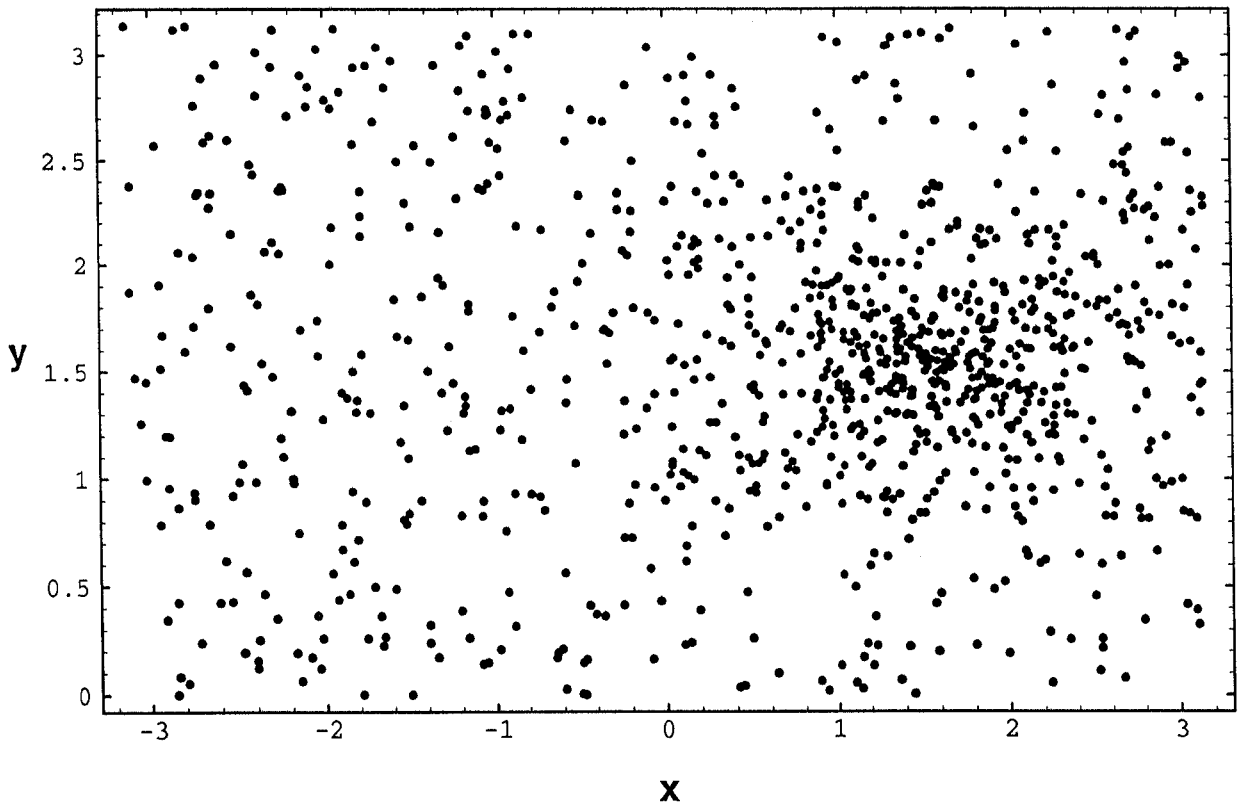
- Here is the progression of sampling through the four steps, showing the contraction of the factor space.

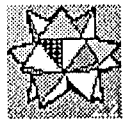




Example Problem -- Enhanced Multi-Stage Monte Carlo Solution (Con't)

- Here is the final plot in full size.





Conclusions

- For certain problems the Multi-Stage Monte Carlo algorithm improves the search for optimum function responses by contracting the factor space searched around the current optimum response point.

- With the decreased sampling density per iteration the probability for focusing on a local function optimum and missing the global optimum is increased.

- The Enhanced Multi-Stage Monte Carlo algorithm appears to decrease the chance of straying to a local optimum by increasing the initial iteration sampling density. Testing so far suggests no significant loss of accuracy compared to Multi-Stage Monte Carlo.